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Question Paper Code : 40262

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fifth Semester

Electronics and Communication Engineering

MA 1251 — NUMERICAL METHODS

(Common to Information Technology)

(Regulations 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State fixed point theorem.

2. Define diagonally dominant system.

3. Compare the Lagrange's interpolation formula and Newton's forward difference formula.

4. Show that $\nabla^3 y_3 = \Delta^3 y_0$.

5. Write down the formula to get first and second order derivatives using Newton's backward difference at $x = x_n$.

6. State two point Gaussian quadrature formula for integration.

7. Given $y' = x + y$, $y(0) = 1$ find $y(0.1)$ by Euler's method.

8. Write down Milne's predictor and corrector algorithm.

9. State the implicit formula to solve the one dimensional heat equation $u_t = \alpha^2 u_{xx}$.

10. For what value of λ, the explicit method of solving the hyperbolic equation $u_{tt} = c^2 u_{xx}$ is stable, where $\lambda = c\dfrac{\Delta t}{\Delta x}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the positive real root of $\cos x - 3x + 1 = 0$ by fixed point iterative method, correct to three places of decimals. (8)

(ii) Find the inverse of the matrix by Gauss-Jordan method $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. (8)

Or

(b) (i) Solve the given system of equations by Gauss-Seidel method (8)

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72.$$

(ii) Find the largest eigenvalue of the matrix $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and the corresponding eigenvector by power method. (8)

12. (a) (i) Find $f(8)$ by Newton's divided difference formulae for the data : (8)

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

(ii) Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for (8)

x :	0	1	2	5
$y = f(x)$:	2	3	12	147

Or

(b) (i) Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence find $f(5)$. (8)

x :	4	6	8	10
$f(x)$:	1	3	8	10

(ii) Obtain the cubic spline approximation for the function $y = f(x)$ from the following data, given that $y_0'' = y_3'' = 0$. (8)

x :	–1	0	1	2
$y = f(x)$:	–1	1	3	35

13. (a) (i) Apply Romberg's method to evaluate $\int_4^{5.2} \log_e x\,dx$ given that (8)

x :	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$\log_e x$:	1.3863	1.4351	1.4816	1.526	1.5686	1.6094	1.6486

(ii) From the table given below, find $f'(30)$ by Newton's forward difference formula for first derivative : (8)

x :	30	31	32	33	34	35	36
$f(x)$:	85.90	86.85	87.73	88.64	89.52	90.37	91.1

Or

(b) Evaluate $\int_2^{2.6} \int_4^{4.4} \frac{dx\,dy}{xy}$ using Trapezoidal rule taking $h = 0.2$, $k = 0.3$. (16)

14. (a) (i) Using Taylor's series method compute $y(0.1)$, $y(0.2)$ if $y' = 1 - 2xy$, $y(0) = 0$. (8)

(ii) Using R-K method of 4th order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given $y(0) = 1$ at $x = 0.2$. Take $h = 0.2$. (8)

Or

(b) (i) Given $y'' + xy' + y = 0$; $y(0) = 1$, $y'(0) = 0$ find the value of $y(0.1)$ by R-K method of 4th order. (8)

(ii) Given $\frac{dy}{dx} = \frac{(1 + x^2)y^2}{2}$, $y(0) = 1$, $y(.1) = 1.06$, $y(.2) = 1.12$, $y(.3) = 1.21$. Compute $y(.4)$ by Milne's predictor corrector formula. (8)

15. (a) (i) Derive a finite difference scheme for solving a Poisson equation. (8)

(ii) Given the wave equation $u_{tt} = u_{xx}$, $0 < x < 1$, $t > 0$ subject to the boundary conditions $u(0,t) = 0$, $u(1, t) = 0$ for $t > 0$ and the initial conditions $u(x,0) = x - x^2$, $u_t(x,0) = 0$ for $0 \le x \le 1$ by taking $h = k = 1/4$, compute the solution for the first 4 time steps. (8)

Or

(b) Use Crank-Nicolson scheme to find the solution of the following initial boundary value problem for one time step : $T_t = T_{xx}$, $0 < x < 1$, $t > 0$ subject to the initial condition $T(x,0) = x - x^2$ for $0 \le x \le 1$ and the boundary conditions $T(0,t) = T(1, t) = 0$ for $t > 0$. Compute the solution by taking $h = 1/4$ and $k = 0.025$. (16)
